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LA-UR--87-2892

DE87 014746

TITLE PHYSICS OF FEW-BODY Λ HYPERNUCLEI

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SUBMITTED TO To be submitted to Nuclear Physics A; Conference
proceedings for the International Symposium on
Strangeness in Nuclear Matter, June 1-5, 1987,
Ban Honnef, West Germany**DISCLAIMER**

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PHYSICS OF FEW-BODY Λ HYPERNUCLEI

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The energies of the particle-stable states in few-body Λ hypernuclei are summarized. Other topics reviewed include: the role of the hypertriton in determining the spin dependence of the ΛN force, the role of the hypertriton in three-body force investigations, the effect of medium modifications upon ΛN - ΣN coupling in the $A=4$ isodoublet and the spin dependence of the ΛN force, the importance of exact equation formalisms in interpreting precision data, and the need for a renewed effort to identify and measure the masses of $\Lambda\Lambda$ hyperfragments.

1. INTRODUCTION

Although the first hypernucleus was identified more than 30 years ago¹, it was in the early 1960's that one realized from the systematics of the ground state energies of the s -shell hypernuclei just how different the physics of the strangeness (S) -1 systems was compared with the nonstrange physics found in nature. Because the Λ has isospin 0, the ΛN interaction has no one-pion-exchange tail and does not support a (deuteron-like) bound state². The $A=3$ hypertriton (${}^3_{\Lambda}\text{H}$) is the lightest $S = -1$ multibaryon bound system. However, the binding occurs only because the Λ clings tenuously to the deuteron in almost a molecular type state. The Λ separation energy

$$B_{\Lambda}({}^3_{\Lambda}\text{H}) = B({}^3_{\Lambda}\text{H}) - B({}^2\text{H}),$$

the energy required to remove the Λ from the hypertriton leaving behind the deuteron, was only³

$$B_{\Lambda}({}^3_{\Lambda}\text{H}) = 0.13 \pm .05 \text{ MeV.}$$

[Here, $B({}^2\text{H}) = -E({}^2\text{H}) = 2.225 \text{ MeV.}$] Nonetheless, the pionic weak decay of the ${}^3_{\Lambda}\text{H}$ was used to establish that the spin is $1/2$ and not $3/2$ (the spin of the Λ

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spin-triplet force,⁵ at least in the ${}^3_{\Lambda}\text{H}$ bound state. The clear difference between the Λ separation energies of the $A=4$ isodoublet³

$$B_{\Lambda}({}^4_{\Lambda}\text{He}) - B({}^4_{\Lambda}\text{He}) - B({}^3\text{He}) = 2.39 \pm 0.03 \text{ MeV}$$

and

$$B_{\Lambda}({}^4\text{H}) - B({}^4\text{H}) - B({}^3\text{H}) = 2.04 \pm 0.04 \text{ MeV}$$

indicated that there was a distinct charge-symmetry-breaking component⁶ in the ΛN interaction. That is, the Λp and Λn interactions differ in such a way that the $A=4$ Λ separation energy difference was three times as large as any charge symmetry breaking deduced from the ${}^3\text{H}$ - ${}^3\text{He}$ binding energy difference. Furthermore, the Λ separation energy in ${}^6_{\Lambda}\text{He}$ ³

$$B_{\Lambda}({}^6_{\Lambda}\text{He}) - B({}^6_{\Lambda}\text{He}) - B({}^4\text{He}) = 3.12 \pm 0.02 \text{ MeV}$$

was only about half that estimated from central force potential models that were fitted to the $A=3$ and 4 hypernuclear data and were also consistent with the available Λp bubble chamber scattering data.^{5,7} We have since made some progress in understanding this physics, but there remain puzzles and new ones have developed. In particular, does the double Λ hypernucleus ${}^6_{\Lambda\Lambda}\text{He}$ exist? If so, then it places limits on the mass of the $S = -2$ dibaryon, the "H" particle of Jaffe.⁸

In this review, I will look briefly at the data on few-body Λ -hypernuclei. The intriguing aspects of ${}^3_{\Lambda}\text{H}$ will be discussed. The question of charge symmetry breaking in the isodoublet will be examined. The $1^+ \rightarrow 0^+$ transition in the $A=4$ system and its relationship to the spin dependence of the ΛN force will be explored. The anomalously small binding of ${}^6_{\Lambda}\text{He}$ will be touched upon. The bearing of $\Lambda\Lambda$ -hypernuclei upon, and the existence of the Λ dibaryon will be discussed. Three important aspects of the physics may be summarized as follows:

1) An improved measurement of the ${}^3_{\Lambda}\text{H}$ binding energy is needed to constrain the models of the hyperon-nucleon interaction. New low-energy data on Λp scattering from tagged Λ beams in $\text{pp} \rightarrow \Lambda\Lambda$ production are anxiously awaited as are data on Λn scattering from $\text{K}^+ \text{d} \rightarrow \Lambda\text{n}\gamma$.

2) ΛN - ΣN coupling is more important in hypernuclear physics than NN - NA coupling in nuclear physics, because the Λ - Σ mass difference is only 80 MeV and the Σ is narrow. This produces a complex spin-dependence of the ΛN

... of the nuclear core state. A simple spin-dependent AN interaction modeled on the free space scattering data, which provides a successful approach to describing the NN interaction in nuclei, will fail. Medium corrections are more important in hypernuclei.

3) A new effort to identify $\Lambda\Lambda$ hypernuclei is needed. If their existence is confirmed, mass measurements for the lightest such hyperfragments would provide severe constraints upon the mass of any possible $S = -2$ dibaryon.

2. S-SHELL DATA SUMMARY

The experimental information available for particle-stable states of the 1s-shell hypernuclei are summarized^{3,9-10} in Table I, where Λ separation energies

$$B_{\Lambda}(\Lambda A) = B(\Lambda A) - B(A-1)$$

and $\Lambda\Lambda$ separation energies

$$B_{\Lambda\Lambda}(\Lambda\Lambda A) = B(\Lambda\Lambda A) - B(A-2)$$

are given. The uncertainty in $B_{\Lambda}({}^3\text{H})$ is fractionally large. It was difficult to extract from emulsion experiments, because of the small binding energy. The value of $B_{\Lambda}({}^6\text{He})$ was determined most reliably because of the available statistics. It was the most common light hyperfragment formed in emulsion studies. The photon energies for the $A=4$ isodoublet were determined by coincidence measurements. They provide a real test of our ability to model the mass 4 system. That is, unlike the nuclear case, here we have two particle-stable states in the same nucleus for which we can numerically solve the set of exact equations that describe the system of strongly interacting baryons within a nonrelativistic, Hamiltonian model framework.

Table 1. Ground-state Λ and $\Lambda\Lambda$ separation energies plus excitation energies of particle-stable states for 1s shell hypernuclei.

	$B_{\Lambda}(\text{MeV})^3$	$E_{\gamma}(\text{MeV})^9$
$\begin{smallmatrix} {}^3\text{H} \\ \Lambda \end{smallmatrix}$	$0.13 \pm .05$	
$\begin{smallmatrix} {}^4\text{H} \\ \Lambda \end{smallmatrix}$	$2.04 \pm .04$	1.04
$\begin{smallmatrix} {}^4\text{He} \\ \Lambda \end{smallmatrix}$	$2.39 \pm .03$	1.15
$\begin{smallmatrix} {}^6\text{He} \\ \Lambda \end{smallmatrix}$	$3.12 \pm .02$	
	$B_{\Lambda\Lambda}(\text{MeV})^{10}$	
$\begin{smallmatrix} {}^6\text{He} \\ \Lambda\Lambda \end{smallmatrix}$	10.6 (?)	

There was some controversy about the interpretation of the emulsion event¹⁰ that was identified as the decay of ${}_{\Lambda\Lambda}^6\text{He}$. A second event¹¹ was reported which corresponds to ${}_{\Lambda\Lambda}^{10}\text{Be}$. The ${}_{\Lambda\Lambda}^{10}\text{Be}$ event ($B_{\Lambda\Lambda} \approx 18$ MeV) has been thoroughly checked and seems reasonably well established. Cluster model calculations indicate that these two $\Lambda\Lambda$ separation energies are consistent within a potential model framework based upon Λ - α and α - α potentials that reproduce the binding energies of ${}_{\Lambda}^6\text{He}$ and ${}_{\Lambda}^9\text{Be}$; that is, the same $\Lambda\Lambda$ model force agrees approximately with the quoted $\Lambda\Lambda$ separation energies for $A=6$ and 10 .

As we shall see below, new experiments to improve our knowledge of the ${}_{\Lambda}^3\text{H}$ binding and to confirm the existence of $\Lambda\Lambda$ hypernuclei are needed.

3. THE HYPERTRITON AND RELATED ISSUES

Because the Λ has spin $J^{\pi} = \frac{1}{2}^{+}$, it can couple to the spin-1 deuteron to form either spin- $\frac{1}{2}$ or $\frac{3}{2}$ Λnp states. It is clear that in the $J = \frac{3}{2}$ system that all two-body interactions must be spin triplet. The $J = \frac{1}{2}$ system is dominated by the spin-singlet ΛN interaction.^{5,12} (The np interaction is a spin triplet, corresponding to a deuteron). That is, one finds

$$J^{\pi} = \frac{3}{2}^{+} : V_{\Lambda N} = V_{\Lambda N}^t$$

$$J^{\pi} = \frac{1}{2}^{+} : V_{\Lambda N} = \frac{1}{4} V_{\Lambda N}^t + \frac{3}{4} V_{\Lambda N}^s$$

A direct analysis of available Λp bubble chamber scattering data^{13,14} cannot determine which of the interactions (singlet or triplet) is the stronger. The extracted scattering lengths are, in fact, highly correlated.¹³ However, it was deduced⁴ from the pionic weak decay of ${}_{\Lambda}^3\text{H}$ that the ground state had $J^{\pi} = \frac{1}{2}^{+}$. Thus, $V_{\Lambda N}^s$ is stronger than $V_{\Lambda N}^t$ in the ${}_{\Lambda}^3\text{H}$ system. Correspondingly, it was concluded that for the scattering lengths

$$|a_{\Lambda p}^s| > |a_{\Lambda p}^t|$$

(Recall that in the nuclear physics convention $a < 0$ implies that there is no two body bound state.) We shall return to this point in the $A=4$ discussion.

An observation related to the statement that the hypertriton corresponds to a Λ bound to the deuteron is that Λnn is not bound. That is, the Λn interaction is not strong enough to hold either the unbound np -singlet or nn system together. In fact, even the Λnn system is unbound in model calculations which limit the strength of the $\Lambda\Lambda$ interaction to be no more than

that of the ΛN interaction. Thus, the ΛN force is a relatively feeble one. This is the result of their being no one-pion-exchange contribution to the interaction. (The Λ has isospin zero, so that the ΛN system cannot simply exchange an isospin-1 pion.) Because of this the ΛN tensor force, which comes in lowest order from \bar{K} and \bar{K}^* exchange, is also not large. Holinde will discuss these details in his presentation.¹⁵

An interesting corollary to the lack of binding in the Λnn system is that the $\Sigma^+ nn$ system is also unbound. The ΣN interaction is even weaker than the ΛN interaction.¹⁶ That is unfortunate, because a bound $\Sigma^+ nn$ system would be unable to decay into ΛNN due to charge conservation.

We shall see in the $A=4$ discussion that charge symmetry breaking in the ${}^4_\Lambda\text{He} - {}^4_\Lambda\text{H}$ ground state binding energy difference

$$\Delta B_\Lambda(0^+) = 2.39 - 2.04 = 0.35 \text{ MeV}$$

is magnified in comparison to the charge symmetry breaking deduced from the ${}^3\text{H} - {}^3\text{He}$ binding energy difference after correcting for the Coulomb interaction of the two protons in ${}^3\text{He}$

$$\Delta B^{\text{CSB}} = [B({}^3\text{H}) - B({}^3\text{He})] - E_C = 0.76 - 0.64 = 0.12 \text{ MeV}.$$

Similarly, coupled-channel effects ($\Lambda N \leftrightarrow \Sigma N$ conversion), or in another language three-body force (ΛNN) effects when the ΣN channel is formally eliminated, are magnified in ${}^3_\Lambda\text{H}$ compared to $NN \leftrightarrow N\Lambda$ coupled-channel (or NNN three-body force) effects in the triton, because the hyperon mass difference $m_\Sigma - m_\Lambda = 80 \text{ MeV}$ is much smaller than $m_\Delta - m_N$.

To make this clear, let us recall that the coupled-channel interaction

$$V_{YN} = \begin{pmatrix} V_{\Lambda N} & V_{\Sigma N} \\ V_{\Lambda N} & V_{\Sigma N} \end{pmatrix}$$

leads to the "box diagram" in a one-channel formalism, when the ΣN channel is formally eliminated by iterating the coupled equations:

$$V_{\Lambda N} = V_{\Lambda N} + V_{\Sigma N} \frac{1}{H_{\Sigma N} - E + \Delta m} V_{\Sigma N}.$$

Schematically this is described in Fig. 1, where in the second term the Λ converts to a Σ (through the transition potential $V_{\Sigma N}$) and then back into a Λ .

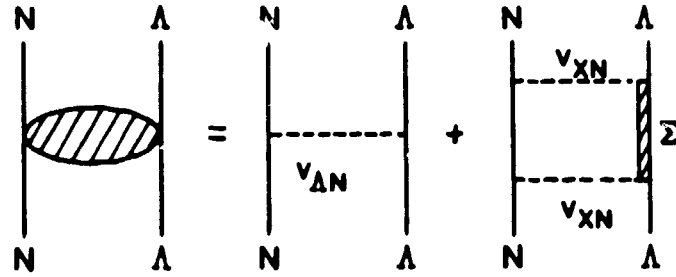


FIGURE 1

Schematic picture of the AN interaction at the box diagram level of including $AN \rightarrow \Sigma N$ coupling

Holinde will emphasize the importance of including the box diagram in one-channel models of the AN interaction.¹⁵ Note that the box diagram is attractive. Note also that it is energy dependent, an effect often neglected in nuclear calculations.

When one embeds such a coupled-channel interaction in the three-body problem, one is led immediately to three-body forces of two types, when the ΣN channel is eliminated. This is illustrated in Fig. 2. Diagram 2(a) describes the simple v_{AN} interaction between the Λ and one of the nucleons. Diagram 2(b) corresponds to the box diagram of Fig. 1. However, the energy denominator now includes the kinetic energy of the second, or spectator, nucleon which weakens its contribution. This is referred to as the dispersive three-body force in the literature, a repulsive energy dependence in the NN force arising from modification of the interaction in the medium. Diagram 2(c) describes the more conventional three-body force, that resulting when all three baryons are directly involved in the interaction.

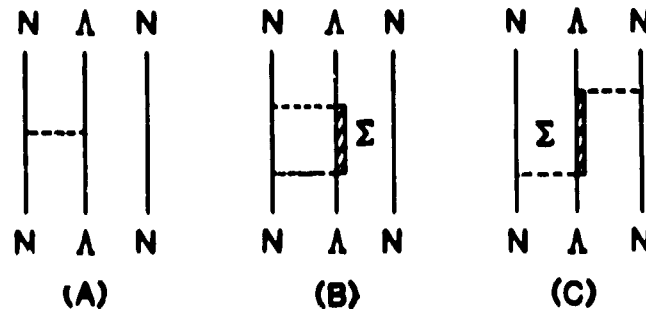


FIGURE 2

Schematic picture of three contributions to the AN interaction in the hypertriton when $AN \rightarrow \Sigma N$ coupling is allowed: (A) direct AN two-body interaction; (B) medium modification of the two-body interaction; (C) conventional three-body interaction

Because the isospin of the Λ ($T=0$) and the Σ ($T=1$) differ, the $\Lambda \leftrightarrow \Sigma$ conversion alters the isospin (and therefore spin) of the interacting nucleon-nucleon pair.¹⁷ Schematically one has two types of terms in the hypertriton wave function:

$$[\Lambda \otimes d]^{T=0} + [\Sigma \otimes d^*]^{T=0}.$$

The deuteron (d) has $T=0$ as does the Λ , so that the spin-singlet NN state (d^* , $T=1$) must couple to the Σ ($T=1$) in order to contribute to the $T=0$ ${}^3\Lambda\text{H}$. Hence, $\Lambda \leftrightarrow \Sigma$ conversion forces a recoupling of the np pair. Because the $(np)^{T=1}$ interaction has an antibound state at about 60 keV, the excitation energy appearing in the energy denominator is only a little more than 2 MeV, and such a medium correction does not quench the attraction from diagram 2(b) significantly in this system.

Three-body force effects (or $\Lambda N \leftrightarrow \Sigma N$ coupling) are clearly evident in the few-body hypernuclear binding energies. Bodmer and Usmani¹⁸ have been developing a model in which ΛN and ΛNN forces are parameterized by analyzing Λp scattering plus the binding energies of the $\Lambda = 3, 4, 4^*, \text{ and } 5$ hypernuclear states. They find a three-body force contribution to be indispensable in reproducing the data. (In contrast, Shinamura¹⁹ reports a fit to just the binding energies which involves only phenomenological ΛN forces. His extracted ΛN potentials disagree markedly with the Λp scattering data, which supports Bodmer's finding that, if one adopts a formalism in which explicit Σ degrees of freedom are eliminated, then ΛNN three-body forces are essential.) The Bodmer analysis contains effectively four potential strength parameters: (1) that of the ΛN interaction combination $\frac{3}{4}V_{\Lambda N}^t + \frac{1}{4}V_{\Lambda N}^s$ found in Λp scattering and in ${}^6\Lambda\text{He}$; (2) that due to the ΛN interaction spin dependence $V_{\Lambda N}^t - V_{\Lambda N}^s$; (3) that of the dispersive diagram 2(b); that of the long-range attractive ΛNN force of diagram 2(c). The spin dependence of the ΛN force is ill determined, due primarily to the lack of precision in our knowledge of $B({}^3\Lambda\text{H})$. Furthermore, the dispersive term appears to dominate the required ΛNN three-body force.

The latter result appears to disagree to some extent with a similar analysis of ${}^3\text{H}$, ${}^4\text{He}$, and nuclear matter by Pandharipande and coworkers, who find the contribution corresponding to diagram 2(c) to be about 1 MeV larger (more attractive) than the dispersive (repulsive) term.²⁰ We shall return to this in the next section. Also, Sauer finds in the Hanover approach, which models the three-nucleon force in terms of $NN \rightarrow N\Delta$ coupling, that the repulsive contribution of the dispersive diagram is slightly smaller than the attractive two-pion-exchange three-body force in the triton.²¹ The most complete

calculations using a realistic coupled-channel interaction are simple separable potential calculations by Dabrowski and Fedorynski.²² Although the calculations were not designed to provide quantitative binding energy estimates, they showed that $\Lambda N \leftrightarrow \Sigma N$ conversion could enhance the model ${}^3_{\Lambda}\text{H}$ binding energy by as much as 200 keV. Because $B_{\Lambda}({}^3_{\Lambda}\text{H})$ is so small, one cannot neglect such effects without further detailed investigation.

Because the hypertriton is loosely bound, it is an ideal laboratory in which to study three-body forces.²³ Here one is relatively insensitive to ill defined short-range effects such as heavy meson exchange. The long-range, two-pion-exchange component of the three-body force ($\Lambda N \leftrightarrow \Sigma N$ conversion can occur by one pion exchange) will dominate. However, a significant improvement in the precision of the ${}^3_{\Lambda}\text{H}$ binding energy as well as improved ΛN scattering data are required before such an investigation can be made quantitative. One must have improved constraints on the realistic hyperon-nucleon two-body potential models. A step in that direction using tagged Λ beams from the CERN $p\bar{p} \rightarrow \bar{\Lambda}\Lambda$ reaction appears realizable.²⁴ In addition, the $K^- d \rightarrow \Lambda n \gamma$ reaction proposed as a means of obtaining information about the low-energy Λn scattering parameters²⁵ is being tested for feasibility²⁶ at BNL.

4. THE Λ -4 ISODOUBLET

The 0^+ ground states and 1^+ spin-flip excited states of the mass 4 hypernuclear isodoublet are shown in Fig. 3 in terms of their Λ separation energies. Because one defines

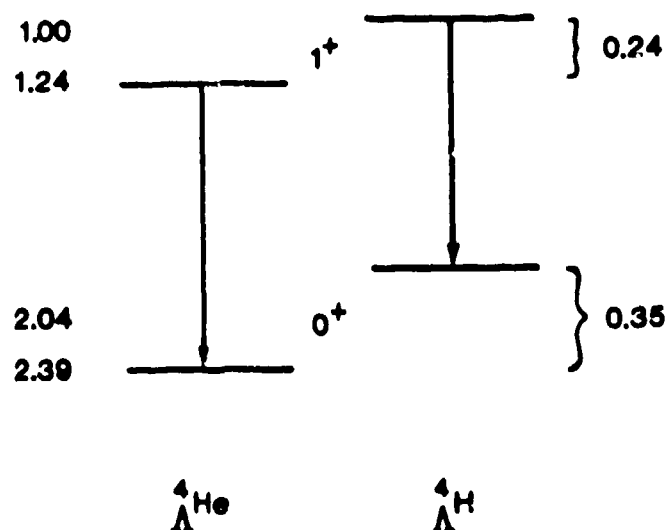


FIGURE 3

Level diagram for the mass 4 isodoublet in terms of Λ separation energies

and

$$B_{\Lambda}({}^4_{\Lambda}\text{He}) = B({}^4_{\Lambda}\text{He}) - B({}^3\text{He})$$

for both the ground states (4) and excited state (4^*), the repulsive Coulomb energy in ${}^3\text{He}$ and ${}^4_{\Lambda}\text{He}$ or ${}^4_{\Lambda}\text{He}^*$ cancels²⁷ to first order. Therefore, as noted previously, charge symmetry breaking in the hypernuclear ground states due to a difference between the Λp and Λn interactions (${}^4_{\Lambda}\text{He}$ has one more Λp interaction and one less Λn interaction than does ${}^4_{\Lambda}\text{H}$),

$$\Delta B_{\Lambda}(0^+) = B_{\Lambda}({}^4_{\Lambda}\text{He}) - B_{\Lambda}({}^4_{\Lambda}\text{H}) = 350 \text{ keV},$$

is three times larger than (and of opposite sign to) the charge symmetry breaking (due to differences between the nn and pp strong interactions) deduced from the experimental binding energy difference in the ${}^3\text{He}$ - ${}^3\text{H}$ nuclear isodoublet. Correcting for the repulsive Coulomb interaction between the two protons in ${}^3\text{He}$, one obtains²⁸

$$\Delta B^{\text{CSB}} = [B({}^3\text{H}) - B({}^3\text{He})] - E_C = 120 \text{ keV}.$$

There is a small Coulomb correction to ΔB_{Λ} , because the Coulomb energy in ${}^4_{\Lambda}\text{He}$ is expected to be larger (more repulsive) than that occurring in ${}^3\text{He}$. This effect actually increases ΔB_{Λ} . It has been estimated to be around 20 keV,^{27,28} yielding a charge-symmetry-breaking energy difference due to the strong interaction of

$$\Delta B_{\Lambda}^{\text{CSB}}(0^+) = 0.37 \text{ MeV}.$$

A charge-symmetry-breaking effect of some type is expected in Λ hypernuclei, because of the significant $\Lambda N \leftrightarrow \Sigma N$ coupling in the hyperon-nucleon interaction.⁶ For example, the Σ^+ and Σ^- masses differ by some 10 MeV, and Λp couples to $\Sigma^+ n$ whereas Λn couples to $\Sigma^- p$. Effects of this ilk have been included in the commendable effort of the Nijmegen group to construct realistic meson-theoretical potential models of the hyperon-nucleon interaction.^{16,29,30} In particular, it has been demonstrated^{31,32} in a model calculation using separable potentials fitted to the low-energy scattering parameters (a, r_0) of the Nijmegen model D¹⁶ that the charge symmetry breaking exhibited by that potential ($V_{\Lambda p}^t = V_{\Lambda n}^t$, $V_{\Lambda p}^s = V_{\Lambda n}^s$) is sufficient to account

for such a value of $\Delta B_{\Lambda}^{\sim\sim}(0^+)$ if one uses a true four-body formalism. That is, one must solve exact four-body equations. (It was also shown in that analysis that a folding model prescription using the same potentials yielded a value of $\Delta B_{\Lambda}^{\text{CSB}}(0^+)$ too small by a factor of 2.) The charge symmetry breaking evidenced by $\Delta B_{\Lambda}(1^+)$,

$$\Delta B_{\Lambda}(1^+) = B_{\Lambda}(^4\text{He}^*) - B_{\Lambda}(^4\text{H}^*) = 240 \text{ keV},$$

has yet to be analyzed in terms of exact four-body equations.

4.1. The $0^+ \leftrightarrow 1^+$ Transition

The fact that there exist two particle-stable states in the $A=4$ isodoublet provides us with a unique opportunity to test our models of the hyperon-nucleon forces. Generating both the 0^+ ground state and 1^+ excited state within the same model is not a trivial exercise, if one is required to utilize forces that reproduce the low-energy properties of the YN scattering data. (Such a test of our ability to model the nonstrange few-body nuclei in terms of the NN interaction does not exist.) An analysis of the structure of the four-body wave function amplitudes generated by solving the Faddeev-Yakubovsky-like exact equations shows^{31,33} that the spin dependence of the two states is not as simple as one might naively expect.

If one neglects spin in this four-body system, the five types of amplitudes that comprise the Schrödinger wave function of either state are depicted schematically in Fig. 4. There are three amplitudes having $[3,1]$ symmetry, i.e., they correspond to configurations in which one baryon is removed from the remaining three. Amplitude A describes a Λ coupled to a three-nucleon core (not necessarily a trinucleon ground state), while amplitudes B and C describe an N coupled to the two types of amplitudes that one finds in the

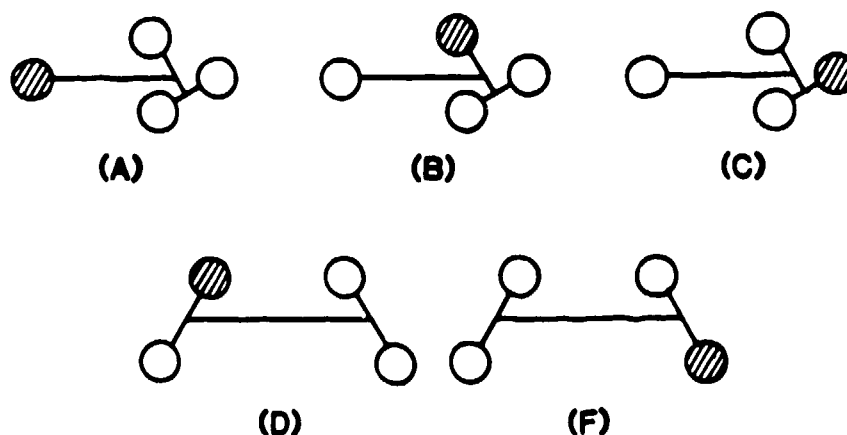


FIGURE 4

Schematic representation of the five amplitudes that determine the $A=4$ wave function in the spin-independent limit of the separable potential equations

coupled to an interacting AN pair. The amplitudes D and F have [2,2] symmetry and describe the decomposition into asymptotic states in which different pairs of baryons interact.

When one includes spin, the number of amplitudes in the 0^+ state expands to 10, while there are 15 amplitudes in the 1^+ state. In a central force approximation, the 0^+ state involves spin-singlet and spin-triplet interacting pairs in the three-body subsystems, but the total spin of the three-body core states can be at most $1/2$ because they must couple to the fourth baryon to form the spin-0 four-body state. The 1^+ state involves the same spin-singlet and spin-triplet interacting pairs in the three-body subsystems, but the three-body core states can have a spin of $3/2$ as well as $1/2$ and still couple to the fourth baryon to form the spin-1 four-body state.

Approximating either the 0^+ or the 1^+ state by $[\Lambda \otimes {}^3\text{H}]^J$ is inadequate. In the case of ${}^4\text{He}$ where the number of amplitudes reduces to two (A and D) because one is dealing with four identical nucleons, neglecting the [2,2] or D amplitude causes one to miss $1/3$ of the α -particle binding energy,³⁴ clearly an unacceptable approximation. ${}^4\text{He}$ is not just composed of states with structure like $[p \otimes {}^3\text{H}]$ and $[n \otimes {}^3\text{He}]$ but contains important elements of the $[d \otimes d]$ and $[d^* \otimes d^*]$ form. Similarly, ${}^4_\Lambda\text{H}$ is not just $[\Lambda \otimes {}^3\text{H}]$ in either the 0^+ or 1^+ state. In the model calculations to be discussed, all five types of amplitudes are coupled to one another. Although the A amplitude is the largest, the two other [3,1] amplitudes and the two [2,2] amplitudes (that is, B, C, D, and F) are each of the order of 10% of A.

In a naive analysis that approximates ${}^4_\Lambda\text{H}$ states as $[\Lambda \otimes {}^3\text{H}]^J$ (that is, a picture in which one retains only the A type amplitudes), one might argue that the $[1/2 \otimes 3/2]^1$ amplitude which contributes to the 1^+ state can be neglected, because the $J=3/2$ excited states of the trinucleon "core" lie far enough above the $J=1/2$ ground state. If so, then the 0^+ state would contain two $[1/2 \otimes 1/2]^0$ amplitudes (with spin-singlet and spin-triplet interacting pairs in the trinucleon) and the model 1^+ state would contain a similar two $[1/2 \otimes 1/2]^1$ amplitudes, which is the origin of the assumption that the 0^+ and 1^+ ${}^4_\Lambda\text{H}$ states are related by a simple spin-flip transition. However, the argument clearly fails when one cannot neglect the remaining amplitudes, which the exact four-body equations require be included. The B and C amplitudes contain three-body "core" states of the hypertriton system. The hypertriton spin- $1/2$ and spin- $3/2$ states are nearly degenerate, and the $J=3/2$ states cannot be neglected. Thus, any simple model analysis of the $A=4$ hypernuclear 1^+ states as just a Λ spin-flip imposed upon the 0^+ state structure can be highly misleading. Furthermore, we shall see that $\text{AN} \rightarrow \Sigma\text{N}$ coupling is important, because the Λ

core states. This was clear in the analysis of NN states that contribute to ${}^3_{\Lambda}\text{H}$ when $\Lambda \rightarrow \Sigma$ conversion is included. Bodmer¹⁷ suggested this as an explanation of the anomalously small value of $B_{\Lambda\Lambda}({}^6\text{He})$. That is, the Σ can couple only to the highly excited T=1, even parity states of ${}^4\text{He}$ which lie some 40 MeV or higher in the spectrum,³⁵ because both the Λ and the ${}^4\text{He}$ core of ${}^6_{\Lambda}\text{He}$ have isospin 0.

4.2. A 0^+-1^+ Model Problem

To illustrate the importance of treating this Λ -4 system in terms of a correct (exact equation) formalism, we consider the following model calculations. We use the Stepien-Rudza and Wycech separable potential approximation³⁶ to the Nijmegen VN coupled channel (ΛN - ΣN) potential model F.³⁰ We include the $\Lambda \rightarrow \Sigma$ conversion effects only implicitly. In other words, the two-channel potentials of ref. 36 are replaced by one-channel effective ΛN potentials which have identical low-energy scattering parameters (scattering length and effective range). The Λ -4 binding energies that result from solving the full set of 10 and 15 coupled, two-dimensional integral equations that describe the 0^+ and 1^+ Λ -4 isodoublet states are:³³

$$B(0^+) = 10.7 \text{ MeV}$$

and

$$B(1^+) = 11.7 \text{ MeV}.$$

The states are inversely ordered with respect to experimental observation. In this approximation of using the free ΛN scattering potentials in the exact Λ -4 equations, the 1^+ state is more bound than the 0^+ state.

The reason is understandable. For two attractive potentials that do not support a bound state

$$|a| > |a'| \rightarrow V \text{ is more attractive than } V'$$

and

$$r_0 > r'_0 \rightarrow V \text{ is more attractive than } V'.$$

If the potential does support a bound state, then

$$a < a' \rightarrow B_2 > B'_2$$

$$r_0 > r'_0 \rightarrow B_2 > B'_2.$$

(As a potential becomes more attractive, the scattering length a approaches $+\infty$ where it just supports a bound state, and then a falls from $+\infty$ as the attractive nature of the potential is further enhanced.) For those who prefer to think of a simple square well, increasing the depth (strength) or the size of the well makes it more attractive for the two-body system. However, the same does not hold true for n -body systems^{37,34} where $n > 2$. Given two attractive potentials V and V' with scattering strengths a and a' and effective ranges r_0 and r'_0 , then one can demonstrate that the binding energies B_n in various systems are related to the potentials as follows. Holding the effective range fixed, then one finds that

$$a < a' \rightarrow B_n > B'_n, n = 2, 3, 4, \dots$$

That is, the binding energy B_n due to potential V is greater than that due to V' in a 2-, 3-, 4-, ... body system. (For an attractive potential that does not support a bound state, $|a| > |a'|$ means that V is more attractive than V' , or closer to supporting a bound state, and $|a| > |a'| \rightarrow B_n > B'_n, n = 3, 4, \dots$) Because the scattering length is related to a volume integral of the potential, this result is expected intuitively. However, when one fixes the scattering length and varies the effective range, then one finds

$$r_0 > r'_0 \rightarrow B_2 > B'_2$$

but

$$r_0 > r'_0 \rightarrow B_n < B'_n, n = 3, 4, \dots$$

That is, as the effective range is increased, the potential is more attractive in the two-body sense, but less attractive in many-body systems. (A variational model calculation illustrating this effect was, in fact, put forth by Thomas in 1935 as an argument for why the nuclear force had to be of nonzero range^{38,39} -- otherwise, the triton would collapse to a point nucleus.) Thus, a mean-field, effective two-body model approximation to an n -body system may lead to an incorrect interpretation of precision experimental measurements. Exact calculations can reveal novel aspects of physics which cannot be obtained in any approximate theory that reduces the calculation to one of an effective two-body equation.

This is illustrated by the AN potential parameters quoted in Table II. The scattering lengths and effective ranges are those of the separable potential model approximation³⁶ to the Nijmegen interaction.³⁰ The λ and β are the strength and range of the rank-one separable potential

$$v(p, p') = -\frac{\lambda}{2\mu} g(p) g(p')$$

$$g(p) = (p^2 + \beta^2)^{-1}$$

that reproduces a and r_0 . (Here μ is the reduced mass of the two-body system.) The scattering lengths are approximately the same. The potential differences are contained in the effective ranges. The effective spin averages that correspond to the two states are⁵

$$0^+: \frac{1}{2}v_{AN}^s + \frac{1}{2}v_{AN}^t$$

$$1^+: \frac{1}{6}v_{AN}^s + \frac{5}{6}v_{AN}^t.$$

Thus, the 1^+ state is dominated by the spin-triplet AN interaction. Because $r_0^t < r_0^s$, the 1^+ state is more bound in the four-body calculation than is the 0^+ state. Based upon the above analysis, it is clear that in an effective two-body formalism just the opposite ordering would be found. The spin-triplet force is weaker than the spin-singlet force in a two-body sense. Although having the 0^+ state more bound than the 1^+ state in a mean field, effective two-body model might be pleasing, the physics would be wrong!

One important effect that is missing from this model based upon AN potentials that describe free scattering is the isospin related to $AN \rightarrow \Sigma N$ coupling. Because the Λ ($T=0$) and Σ ($T=1$) couple differently to $T=1/2$ core states that are composites of three $T=1/2$ nucleons than to elementary $T=1/2$ nucleons, there is a significant medium modification of the AN interaction. Neglecting $T=3/2$ trinucleon core states (having excitation energies of some 80

Table II. The potential parameters along with a and r_0 for the free space AN interaction.

	v_{AN}^s	v_{AN}^t
$\lambda(\text{fm}^3)$	0.0952	0.3262
$\beta(\text{fm}^{-1})$	1.2011	1.7251
$a(\text{fm})$	-1.97	-1.95
$r_0(\text{fm})$	3.90	2.43

$$v_{YN}^i = \begin{pmatrix} v_{\Lambda N}^i & v_{XN}^i \\ v_{XN}^i & v_{\Sigma N}^i \end{pmatrix}, \quad i = s, t$$

exhibit altered spin-isospin coefficients for the off-diagonal coupling terms. In particular, the spin-singlet force is modified in the 0^+ state

$$0^+: v_{YN}^s = \begin{pmatrix} v_{\Lambda N}^s & \frac{1}{3} v_{XN}^s \\ \frac{1}{3} v_{XN}^s & v_{\Sigma N}^s \end{pmatrix}$$

and the spin-triplet force is modified in the 1^+ state

$$1^+: v_{YN}^t = \begin{pmatrix} v_{\Lambda N}^t & -\frac{1}{5} v_{XN}^t \\ -\frac{1}{5} v_{XN}^t & v_{\Sigma N}^t \end{pmatrix}.$$

The rank-one separable parameters that reflect these medium modifications are quoted in Table III. Both interactions are weaker ($|a|$ is smaller and r_0 is larger) than those in Table II in the few-body sense. Hence, the binding energies of both states will be reduced. However, the modified spin-triplet interaction, which is combined with the free space spin-singlet interaction in the 1^+ state, has been weakened more (coefficient $-1/5$ compared to $1/3$) than the modified spin-singlet interaction, which is combined with the free space spin-triplet interaction, in the 0^+ state. The model binding energies are³³

$$B(0^+) = 9.6 \text{ MeV}$$

$$B(1^+) = 8.2 \text{ MeV}.$$

Table III. Potential parameters along with a and r_0 for the medium modified interactions.

	$v_{\Lambda N}^s(0^+)$	$v_{\Lambda N}^t(1^+)$
λ	0.0739	0.1814
β	1.1828	1.6061
a	-1.33	-0.95
r_0	4.68	3.50

of both states. However, the 1^- state, dominated by the spin-triplet interaction, suffers the larger change.

The model 0^+-1^+ energy difference now has the correct sign and is

$$E_\gamma = 1.4 \text{ MeV.}$$

This is a model calculation which has neglected other possibly important effects such as tensor forces. However, it does illustrate the important role that $\Lambda N \rightarrow \Sigma N$ coupling plays in understanding the s -shell Λ -hypernuclei and the complex nature of the spin-dependence of the ΛN interaction in the nuclear medium. Furthermore, the spin-singlet ΛN interaction may turn out to appear more attractive than the spin-triplet interaction in few-body bound states but be weaker in the two-body sense in free space. Hypernuclear physics is most interesting.

5. $\Lambda\Lambda$ HYPERFRAGMENTS

Two emulsion events have been reported which were interpreted as $\Lambda\Lambda$ hypernuclei. The ${}^{10}_{\Lambda\Lambda}\text{Be}$ event¹¹ with $B_{\Lambda\Lambda} \approx 18 \text{ MeV}$ was found first and has been rather thoroughly checked. The ${}^6_{\Lambda\Lambda}\text{He}$ event¹⁰ with $B_{\Lambda\Lambda} \approx 10.6 \text{ MeV}$ has been somewhat controversial. The importance of such $\Lambda\Lambda$ hyperfragments is unquestioned. They provide our only window to study the $\Lambda\Lambda$ interaction, and their existence bears upon that of the "H" dibaryon⁸ -- a uuddss spatially symmetric combination of 6 quarks that could take maximal advantage of the strong magnetic-color forces in the one-gluon-exchange interaction among quarks.

Although the interpretation of the ${}^6_{\Lambda\Lambda}\text{He}$ event has not been universally accepted, model calculations seem to indicate that the two $\Lambda\Lambda$ events are consistent.⁴⁰ When $\Lambda\alpha$ forces are parameterized to reproduce the Λ separation energy in ${}^6_\Lambda\text{He}$ and $\alpha\alpha$ forces are parameterized to describe $\alpha\alpha$ scattering and ${}^6\text{Be}$ levels, the $\Lambda\Lambda$ force needed to account for $B_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\text{Be})$ also accounts for the quoted value of $B_{\Lambda\Lambda}({}^6_{\Lambda\Lambda}\text{He})$.

Because one would expect a $\Lambda\Lambda$ hyperfragment to decay quickly into an H dibaryon if the H has a mass smaller than $2m_\Lambda$, the observation of $\Lambda\Lambda$ hyperfragments argues against the existence of an H bound with respect to $\Lambda\Lambda$ decay. Emulsion events are identified by weak decay ($\Lambda \rightarrow N\pi$) of the hypernucleus,⁴ which is strongly suppressed as the mass is increased above $A=5$. Therefore, events in which both Λ s in ${}^{10}_{\Lambda\Lambda}\text{Be}$ decay weakly must be very rare indeed. Many more ${}^{10}_{\Lambda\Lambda}\text{Be}$ hypernuclei must have been formed and decayed undetected by the $\Lambda N \rightarrow NN$ weak process.

for this unique perturbative QCD prediction of the H dibaryon, serious consideration should be given by experimentalists to exploitation of a signature for $\Lambda\Lambda$ hypernuclei other than their pionic decay modes. Renewed efforts to confirm the existence of ${}_{\Lambda\Lambda}^6\text{He}$ are called for, as is a search for lighter mass $\Lambda\Lambda$ hypernuclei. From $B_{\Lambda\Lambda}({}^{10}\text{Be}) = 18 \text{ MeV}$, one can deduce that

$$m_H \geq 2m_\Lambda - 20 \text{ MeV}.$$

Otherwise, the $\Lambda\Lambda$ pair should decay rapidly into an H. If ${}_{\Lambda\Lambda}^6\text{He}$ is confirmed, then one can surmise that

$$m_H \geq 2m_\Lambda - 10 \text{ MeV}.$$

If ${}_{\Lambda\Lambda}^6\text{He}$ is not confirmed, then one can bound m_H between these two values, because model calculations^{40,27} that are consistent with $B_{\Lambda\Lambda}({}^{10}\text{Be})$ being about 18 MeV also yield an estimate of $B_{\Lambda\Lambda}({}^6\text{He})$ of about 10 MeV -- assuming the $\Lambda\Lambda$ pair do not decay into an H. If ${}_{\Lambda\Lambda}^6\text{He}$ is confirmed, does the hyperfragment ${}_{\Lambda\Lambda}^4\text{H}$ exist? Because ${}^3_\Lambda\text{H}$ binds, ${}_{\Lambda\Lambda}^4\text{H}$ will also bind, if the $\Lambda\Lambda$ pair do not decay into an H. The ${}^4\text{He}(K^-, K^+) {}_{\Lambda\Lambda}^4\text{H}$ reaction is a candidate for the search, although the momentum transfer in such double-strangeness-exchange reactions is not favorable to ground-state formation.

6. SUMMARY

An improved measurement of the ${}^3_\Lambda\text{H}$ binding energy is called for to constrain the spin dependence of the hyperon-nucleon interaction and to test our ability to model three-body forces. Improved low-energy Λp scattering data from tagged Λ beams as well as Λn data from the $K^- d \rightarrow \Lambda n \gamma$ reaction are needed. ΛN - ΣN coupling is an important aspect of hypernuclear physics, because the Λ - Σ mass difference is small and the Σ is narrow. This produces a complex spin dependence for the ΛN interaction that varies with the isospin of the nuclear core state. Medium corrections are more important in hypernuclei than in normal nuclei. This effect is maximal in ${}^6_\Lambda\text{He}$ and can be interpreted in terms of a repulsive three-body (nuclear core dependent) force. A renewed effort to identify $\Lambda\Lambda$ hypernuclei is called for. Mass measurements of the lightest such hyperfragments would provide important constraints on the mass of any $S = -2$ dibaryon.

The work of the author has been supported in part by the U. S. Department of Energy. He gratefully acknowledges the appointment as a Visiting Research Scholar by Flinders University and a long and fruitful collaboration with D. R. Lehman in this area of physics.

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